'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 2nd Semester Examination, 2023

CC3-MATHEMATICS

REAL ANALYSIS

(REVISED SYLLABUS 2023 AND OLD SYLLABUS 2018)

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***four*** questions from the following: 3×4 = 12

(a) Prove or disprove: Every bounded sequence is a Cauchy sequence. 3

(b) Show that $\lim\limits_{n\to\infty}\left(\dfrac{2n!}{(n!)^2}\right)^{1/n} = 4$. 3

(c) Examine if the series $\sum\limits_{n=2}^{\infty}\dfrac{\log n}{\sqrt{n+1}}$ is convergent or not. 3

(d) Show that a finite set is a closed set. 3

(e) Check whether the set $\left\{1, -\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, -\frac{1}{3}, \cdots\cdots\right\}$ is open or closed. 3

(f) Prove that $\log_{10} 5$ is not rational. 3

GROUP-B

Answer any *four* questions from the following 6×4 = 24

2. Prove that the union of two countable set is countable. 6

3. Test the convergence of the series $1+\dfrac{x}{1!}+\dfrac{2^2x^2}{2!}+\dfrac{3^3x^3}{3!}+\dfrac{4^4x^4}{4!}+\cdots\cdots$ for $x>0$. 6

4. Show that every infinite bounded set has a limit point. 6

5. For any two sets X and Y of $\mathbb{R}$, prove that 3+3

(a) $\text{ext}(X\cup Y)=\text{ext}(X)\cap\text{ext}(Y)$

(b) $\text{int}(X\cap Y)=\text{int}(X)\cap\text{int}(Y)$.

6. (a) Prove that $\left\{\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots\cdots+\frac{1}{2n}\right\}_{n\in\mathbb{N}}$ is a convergent sequence. 4+2

(b) Find the upper limit and lower limit of the sequence $\left\{(-1)^n\left(1+\frac{1}{2n}\right)\right\}_{n\in\mathbb{N}}$.

7. Prove that the series 6

$$1+\frac{\alpha}{\beta}+\frac{\alpha(\alpha+1)}{\beta(\beta+1)}+\frac{\alpha(\alpha+1)(\alpha+2)}{\beta(\beta+1)(\beta+2)}+\cdots\cdots$$

where α, β are positive, converges if $\beta>\alpha+1$ and diverges if $\beta\leq\alpha+1$.

GROUP-C

Answer any *two* questions from the following 12×2 = 24

8. (a) Prove that a sequence $\{x_n\}$ converges to l iff both the sub-sequences $\{x_{2n}\}$ and $\{x_{2n-1}\}$ converge to l. 4

(b) Prove that $\lim\limits_{n\to\infty}\left(\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\cdots\cdots+\frac{1}{\sqrt{n^2+n}}\right)=1$. 4

(c) Show that every absolutely convergent series is convergent. 4

9. (a) Find the limit points of the set $S=\{\frac{2}{p}+\frac{3}{q}:\ p, q\in\mathbb{N}\}$. Is the set S closed? Is the set S open? Justify your answer. 4+1+1

(b) Prove that the closure of a set $S\subset\mathbb{R}$ is the smallest closed set containing S. 6

10.(a) If a sequence $\{a_n\}$ is bounded then show that limit inferior and limit superior of $\{a_n\}$ are both finite. 4

(b) If $a_n=\sin\frac{n\pi}{2}+\frac{(-1)^n}{n}$, $n\in\mathbb{N}$, then show that $\underline{\lim}\,a_n=-1$ and $\overline{\lim}\,a_n=1$. 4

(c) Show that the sequence $\{S_n\}$, where $S_n=1+\frac{1}{2}+\frac{1}{3}+\cdots\cdots+\frac{1}{n}$ cannot converge. 4

11.(a) Prove that if $\sum\limits_{n=1}^{\infty}a_n$ be convergent series of positive real numbers, then $\sum\limits_{n=1}^{\infty}a_{2n}$ is convergent. 6

(b) Prove that the series $\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\frac{1}{4^p}+\cdots\cdots$ converges for $p>1$ and diverges for $p\leq 1$. 6

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